Taxes and Income Distribution in Chile: Some Unpleasant Redistributive Arithmetic

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November 30, 1997

Abstract

This paper quantifies the direct impact of taxes on income distribution at the household level in Chile and estimates the distributional effect of several changes in the tax structure. We find that income distributions before and after taxes are very similar (Gini coefficients of 0.4889 and 0.4920, respectively). Moreover, radical modifications of the structure, such as raising the value added tax from 18 to 25% or substituting a 20% flat tax for the present progressive income tax (top marginal rate of 48% for monthly incomes over $6,000) affect the after-tax distribution only slightly.

We present some arithmetic showing that the scope for direct income redistribution through progressivity of the tax system is rather limited. By contrast, for parameter values observed in Chile, and possibly in most developing countries, the targeting of expenditures and the level of the average tax rate are far more important determinants of the income distribution after government transfers. Thus, a high-yield proportional tax can have a far bigger equalizing impact than a low-yield progressive tax. Moreover, a simple model shows that the optimal tax system is biased against progressive taxes and towards proportional taxes—the bias grows with the degree of inequality of pre-tax incomes.

Our results suggest that to reduce income inequality, the focus of discussion should be on the amount to be redistributed, the targeting of public spending, and the relative efficiency of alternative taxes, and *not* on the progressivity of the tax system.

Key words: income distribution, taxes, progressivity.

JEL classification: H22, H24, H29.

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1 Introduction

Income distribution remains one of the most debated economic issues in developing countries, and Chile is no exception.[2] Although poverty has declined fast and steadily during the last ten years, inequality has not changed much. Quite often it is concluded that the stagnation of the income distribution is due to inappropriate policies that should be replaced by direct redistributive measures. Given that one of the ways the state can affect income distribution is through the tax system, there is permanent discussion on the distributional effects of taxes. This discussion heats up whenever the government proposes some tax amendment. For example, whenever it has announced its intention to raise the rate of the Value Added Tax (VAT), a heated debate has ensued over its incidence and distributional impact. On the other hand, many people react with concern when the possibility of reducing the progressivity of income taxes is raised, because they think this will significantly increase income inequality.

The purpose of this paper is to quantify the distributional impact of the Chilean tax system and to assess the sensitivity of the distribution of income to changes in the structure of taxes and rates. We do so by constructing a model that incorporates the main taxes and allowances in place in Chile in 1994. We estimate the true income of individuals[3] with data from the 1994 National Socioeconomic Characterization Survey (CASEN) taken by the Planning Ministry, and "match" this information with taxpayer records kept by the Chilean Internal Revenue Service (SII). In this way we are able to estimate the extent of underreporting of income, as well as deductions for allowances which we impute for each income percentile. At the same time, using data from the Family Budget Survey (EPF) from the National Institute of Statistics (INE), carried out in 1987-88, we estimate the composition of household consumption and the amount of indirect taxes that each household pays.

Like most studies for developed countries, we conclude that the tax system has little effect on income distribution (before- and after-tax Gini coefficients of 0.4889 and 0.4920). We also show that major departures from current tax rates do not alter this conclusion. For example raising the VAT rate from 18% to 25%, or replacing the present income tax (top marginal rate of 48% for monthly incomes over US$6,000) by a flat tax with a uniform marginal rate of 20%, hardly alters the income distribution at all. The data suggests that

[2] For recent studies on income distribution in Chile see Beyer (1996), Contreras (1996) and Cowan and De Gregorio (1996).

[3] The definition of income we use is given in section 2.1.

this is not due to tax loopholes or massive evasion: while around 27% of the theoretical income tax base is not reported, most household incomes, including some from the wealthiest decile, are relatively low.[4] For that reason, although most of income tax revenues come from individuals from households in the wealthiest decile, the average tax rate is low, slightly below 4%. Even if all tax-free allowances and underreporting of income were eliminated, the average rate would not reach 6%. The second conclusion is that the tax system in force in 1994 is slightly regressive. This is because a regressive tax (VAT) is very important, and is only partially compensated by the progressive income tax, which, as we already mentioned, raises little income from the wealthiest decile. This slight regressivity of the Chilean tax system contrasts with most studies of the distribution of tax burdens in developing countries, which find overall tax systems to be broadly progressive.[5]

Motivated by these results we present a simple formalization showing that the scope for direct income redistribution through a progressive tax system is small. Moreover, we also show that progressivity is increasingly ineffective the more *unequal* the pre-tax distribution. By contrast, for parameter values observed in Chile, the targeting of expenditures and the level of the average tax rate are far more important determinants of the income distribution. For example, after accounting for redistribution, the high-yield but slightly regressive VAT reduces inequality far more than the low-yield, strongly progressive income tax.

Of course, if all taxes cost the same to administer, have the same revenue potential and create the same excess burden it would always be better to levy progressive taxes. Nevertheless, in practice the VAT scores better than progressive income taxes on all three counts. We present a simple model showing that when this is so the optimal tax system is biased against progressive taxes and towards proportional taxes. Somewhat surprisingly, this bias is stronger the more *unequal* the pre-tax distribution.

The methodology we use to estimate the distributional impact of the Chilean tax system is based on the standard literature on the distribution of the annual tax burden pioneered by Ockner and Pechman (1974).[6] These studies estimate income and consumption patterns for each household in a given year and calculate the burden on the basis of tax data and a series

[4]By convention, the last decile is the one with highest incomes. However, for our presentation to be fluid and for the reader not to have to think constantly whether the last decile is the poorest or the wealthiest, from now on we will refer to the first decile as the poorest and the last decile as the wealthiest. Although the poorest decile is indeed poor, most of the households in the wealthiest decile are not what in ordinary language would be called rich. The income distribution within the wealthiest decile is summarized in Table 9, presented in section 3.2.

[5]See, for example, Jayasundera (1986) for Sri Lanka, Lovejoy (1963) for Jamaica, Malik and Saquib (1989) for Pakistan, McLure (1971) for Colombia and Sahota (1969) for Brazil.

[6]See also Pechman (1985).

of incidence assumptions.[7] Unlike these studies, we incorporate the multisectoral effects of indirect taxes in greater detail. Using the 1986 National Accounts input-output matrix we estimate the effect of taxes charged on inputs on the tax burden faced by households that consume the final goods incorporating those inputs. Moreover, our data enable us to estimate the number of non-filers and the magnitude of underreporting of the income tax.

Our work updates that of Aninat, Arellano and Foxley (1980), who used a similar methodology to study the distribution of the tax burden under the tax system in force in Chile in 1969, and the study by Schkolnik (1993), who estimated the distribution of the tax burden and government spending at the quintile level in 1990. Access to a series of data sources at the micro level which have not been previously exploited, in particular individual taxpayer records kept at the SII, enable us to work at the level of each income percentile, thereby obtaining more precise estimates of the way the direct tax burden is distributed. Second, the program we construct enables us to estimate the distributive consequences of tax amendments such as changes in rates or alterations to allowances. Third, we present the first estimation of the magnitude of underreporting of incomes in Chile and its distributive impact using detailed and comprehensive microdata.

Before proceeding, we mention the main limitations of our model. In the first place, the calculations assume that changes in the tax system do not affect the composition of spending or production decisions. Therefore, our model does not allow us to assess the welfare effect of the distortions that taxes create, nor how the costs of such distortions are distributed. Incorporating these effects would require a computable general equilibrium model, which goes beyond the scope of this paper.[8] Having said this, the approach we adopt has the virtue of allowing us to work with microeconomic information which is considerably more detailed than what can be incorporated in computable general equilibrium models.[9]

In the second place our income definition is annual. As is well known, annual income is not always a good reflection of permanent income, and this may lead to an exaggeration of both inequality and the regressivity of consumption taxes (see the discussion in Section 3.1).

Third, income figures reported by the CASEN survey for the higher centiles is likely to

[7] A second type of study estimates the income each individual perceives over her lifetime and the total amount of taxes she will pay. The most important study in this line of research is that of Fullerton and Rogers (1993) for the United States, which uses a panel of individuals and determines the incidence of each tax with the help of a computable general equilibrium model.

[8] Studies which use a computable general equilibrium model for the United States are Ballard et al. (1985) (a one year horizon) and Fullerton and Rogers (1993) (intertemporal horizon).

[9] To solve a computable general equilibrium model one needs to iterate repeatedly until an equilibrium set of prices is found; this can only be done by limiting the size of the sample or by aggregating microdata.

be less reliable, for which reason this paper may underestimate the distributive impact of the tax system on these centiles.

Fourth, in some cases the CASEN survey does not allow us to distinguish incomes that should form part of the taxable base from those that are exempt. Thus, part of what we identify as underreporting of income does not correspond to evasion but to income that taxpayers legally do not have to declare. For the same reason it will not be possible to estimate separately the effects of eliminating certain tax-free allowances.

Fifth, we assume that the evasion of indirect taxes (e.g., VAT) only benefits producers. The reason is that the available data does not enable us to estimate the distributional effects of the evasion of indirect taxes, because the CASEN survey does not allow us to identify the owners of firms evading taxes. Finally, and for this same reason, we assume that profits retained by firms are not income for the households that own those firms during the year in which the income accrues. This has two implications: (a) income is probably more concentrated than is suggested either by the CASEN survey or the results we present, and (b) undistributed profits from investment companies, set up to avoid the highest marginal rates of the Global Complementary tax, are not included as household income in our calculations.[10] This, together with the fact that income taxes in Chile are integrated, means that the First Category Tax, which is charged on company profits, has no effect on income distribution.

The rest of the paper is organized as follows. In Section 2 we briefly describe the main features of the Chilean tax system. In section 3 we describe the methodology and the data sources we use. Section 4 estimates the progressivity of the tax system in place in 1994 and shows that the distribution of income is remarkably insensitive to radical modifications of the tax structure. In section 5 we present the arithmetic exercises. Section 6 presents the model. Section 7 summarizes the conclusions.

2 The Chilean tax system: a primer

In this section we briefly describe the main features of the Chilean tax system.

Direct taxes. The main direct tax in Chile is the income tax or *Impuesto a la Renta*. In 1994 it comprised three taxes: (a) A 15% flat business tax (*Primera Categoría*); (b) a progressive wage tax (*Segunda Categoría*) and a progressive general income tax (*Global*

[10] In Section 4 we briefly discuss how our results would be affected by including income from investment companies in our definition of income.

Table 1: The Chilean tax system (1996)

	% of total rev.	**% of GDP**
Direct taxes		
Business tax	12.8	2.3
Personal taxes[a]	7.9	1.5
Inheritances	0.2	0.0
Real estate	4.0	0.8
Foreign corporations	3.4	0.6
State-owned corp.	1.2	0.2
	29.5	**5.4**
Indirect taxes		
VAT	42.2	7.7
Alcohol & tobacco	4.7	0.9
Gas	7.4	1.4
Luxury	1.6	0.3
Import tariffs	10.8	2.0
Bank operations	3.7	0.7
	70.5	**13.0**
Total	**100%**	**18.4**

[a] Net of business tax credit

Source: IRS

Complementario, henceforth GC).[11] On an annual basis, marginal rates and income brackets of the wage and general income tax are the same. However, while the wage tax is paid on a monthly basis, the GC tax is levied on annual income. Both the wage and GC taxes are personal, that is they are levied on individuals and not households.

The main feature of the income tax is that it is integrated. Each year individuals consolidate all their incomes, regardless of their source, into a comprehensive taxbase, and then compute their *total* tax obligation by applying the progressive scale of the GC tax. All business and wage taxes paid on incomes included in the comprehensive taxbase are

[11] See Table 3 for income tax brackets.

then deducted as credits from the GC tax dues. Two features of the income tax imply that it is not *fully* integrated, however. First, profits retained by firms do not enter the GC comprehensive base; correspondingly, credits on the business tax cannot be claimed until profits are paid out. Second, the progressive wage tax is levied on a monthly basis. Those individuals whose only source of income are wages do not file a GC tax return. If their wage income fluctuates from month to month they may end up paying higher taxes than an individual with exactly the same annual income but who receives income from sources different than wages.

There are four major allowances in the income tax. First, Article 57 bis., letters (a) and (b) of the income tax law, which allows GC taxpayers to deduct from their tax base 20% of the amount purchased in newly-issued shares in publicly owned corporations in perpetuity, as well as financial savings in specially designated instruments. Second, an exemption on savings of less than about US$1,000 per year, which benefits taxpayers who pay only the wage tax. Third, an exemption on income arising from properties favored by the Law Decree 2 of 1968 (DFL2). Lastly, unincorporated businesses in several activities (among them agriculture and transport) and small businesses are favored by simplified accounting rules that make most of their income exempt.

In addition, Chilean tax law allows individuals to set up a company, transform part of their personal incomes into business income, compute various expenses as costs, and pay the flat 15% business tax on profits. As long as earnings are not distributed they avoid the highest rates of the GC tax. This enables individuals both to smooth their tax burden and postpone paying the GC tax. Moreover, several schemes allow them to partially avoid the top brackets of the GC tax altogether. For example, relatives in lower brackets may own part of the company, the company may buy assets that are used in personal consumption, or the business can be sold after accumulating profits and be favored by exemptions granted to non-habitual capital gains.

Table 1 shows the yield of each direct tax both as a percentage of total tax revenues and as percentage of GDP. It can be seen that in 1996 direct taxes accounted for 29.5% of total tax revenues (5.4% of GDP). There are two direct taxes that we ignore in this study because we lack the necessary data to allocate them to households: A yearly property tax and an inheritance tax which represent 4.2% of total tax revenues. Moreover, for obvious reasons we do not consider the additional tax paid by foreign corporations when profits are paid out at their headquarters, and the business tax paid by state-owned companies.

Indirect taxes. The main indirect taxes in Chile are a comprehensive value-added tax (VAT) which in 1994 was levied on most transactions at the uniform rate of 18%, a general uniform import tariff of 11%,[12] and a series of excise taxes levied on particular goods like alcohol, tobacco, gasoline and luxury goods (e.g., jewelry, most cars, lotteries and furs). Some goods and services are exempted from VAT, notably professional, educational and health services, transportation, and cultural and sports events. As can be seen from Table 1, indirect taxes account for more than 70% of tax revenues (13% of GDP). In the exercises below we ignore the indirect tax that is levied on bank cheques and credit operations, because we lack the data needed to impute them to households (3.7% of total tax revenues).

3 Concepts, data sources and methodology

3.1 Definition of income

This paper focuses on income distribution at the household level. By "income of a household" we mean the sum of the incomes of each of the members of the household received from work, retirement and survivor pensions, allowances for the disabled, interest paid by firms and financial institutions, profits distributed by firms, consumption of own production, private transfers (e.g., alimony payments and allowances) and imputed income from housing. This definition does not consider any accrued income or government transfers. It therefore excludes: (a) Profits that firms did not distribute during 1994. (b) Government transfers in money or kind. (c) Proceeds from the sale of financial or physical assets. Company profits are excluded because our source of data on income, the CASEN survey, does not identify to whom profits retained by firms belong.[13] For the same reason, we are not able to impute the income received through companies formed to avoid the higher brackets of the GC tax. Government transfers (e.g., family allowances, welfare payments) are not included because our aim is to estimate the distributive impact of the tax system prior to any government redistribution.

We measure income on an annual basis. As is well known, there are several reasons why current income may not be an appropriate measure of the lifetime income of an individual: the individual may be unemployed during the year the survey was taken; some individuals are subject to considerable yearly income fluctuations; incomes vary over the life cycle,

[12] Because of several free trade agreements, imports from some countries pay lower tariffs. Moreover, selected imports also pay antidumping counterveiling duties.

[13] Ockner and Pechman (1974) distribute profits retained by firms in proportion to each individuals dividend income. The 1994 CASEN survey groups income from capital in a single category, without distinguishing between dividends and interest income. Thus, it is not possible to impute retained profits at the individual level.

and towards the end of their lives people tend to consume part of their savings. Thus, for example, a person whose current income is low may have a high permanent income and be intertemporally wealthy. Unfortunately, there is no data in Chile to carry out a study of intertemporal incidence. As regards our aim in this paper, it should be noted that studies which have estimated income distribution over the life cycle in developed countries usually find that inequality is considerably less than that suggested by annual studies.[14] Second, when the definition of intertemporal income is used, saving is not necessarily exempt from consumption taxes. This point is important, for in the next section we find that in Chile VAT is regressive, largely because income saved during a given year does not pay this tax, and the wealthiest deciles save a larger fraction of their incomes. It follows that the regressivity of VAT would be less if one considered an intertemporal framework.

3.2 Data requirements

As we already mentioned, in Chile the income tax is personal, it is levied at the indivual, not at the household level. Therefore, to estimate the distributive impact of the tax system at the household level we need the following information:

- Amount and origin of income actually received by each of the individuals in the household.
- Compliance of each individual, that is the fraction of income that each individual declared.
- Tax-free allowances that benefit each individual.
- Distribution of each households spending on each type of good, as well as its level of spending, so as to estimate the amount that the household pays in each indirect tax.
- The incidence of each tax.

Ideally we would obtain all the data from the same source: taking a sample of households for which one knows each of its income sources, its consumption patterns, the tax-free allowances it was granted and what it paid in terms of each tax. However, in Chile no sample of households with these characteristics is available, so instead we use several sources and

[14] See Chapter 1 of Fullerton and Rogers (1993) for a review of studies of intertemporal incidence.

make assumptions to splice them together.[15] Below we briefly describe our data sources, the assumptions on incidence we make and the methodology we use to calculate taxes. A detailed description is relegated to a rather lengthy Methodological Appendix, available upon request from the authors.

3.3 The data

Incomes. The incomes of each individual and household were estimated with data from the 1994 CASEN survey. This is a biannual survey taken by the Planning Ministry. In 1994 it comprised 178,057 individuals from 45,379 households. The survey separates the income of each individual into its different sources and allows us to identify the household to which the individual belongs. Our calculations assume that each individual reports her true income. Survey data were adjusted by ECLAC[16] so that once the appropriate weights are applied total per capita sample income coincides with the national accounts figures for each income category.[17] Table 2 shows the top, bottom and average monthly per-capita income of households in each income decile, before taxes (all the figures we present are in US dollars of November of 1994).

Direct taxes. The SII database contains data on 312,124 taxpayers of GC and 159,626 taxpayers who pay only the wage tax. These correspond to 1995 income tax returns (fiscal year 1994) and account for virtually all revenues from the GC tax, and half of wage tax revenues.[18] Accordingly, in the case of the wage tax the sample of 159,626 taxpayers who only pay that tax was assumed to be representative of the sample universe. Table 3 shows the number of taxpayers by income bracket as declared to the SII.

Allowances. The amount of tax-free allowances corresponding to Article 57 bis., letters (a) and (b), deducted by each individual was estimated using data from the SII database described in the previous paragraph. In 1994, 37,101 taxpayers took advantage of letter (a) of Article 57 bis., deducting slightly more than US$135 million from their taxable base. A

[15] In their classic study Pechman and Ockner (1974) also splice different data sources. However, the information available allowed them to make more precise splices than those permitted by the data available in Chile.

[16] The United Nation's Economic Comission for Latin America and the Caribbean.

[17] The proportional difference between the CASEN and national account data is imputed uniformly for each income category. The one exception is capital income, which is fully imputed to the top quintile of households.

[18] Due to typing mistakes, information was lost on about one thousand GC taxpayers and 100,000 wage taxpayers. This problem will be corrected in the next version of this paper, which will use data from the CASEN 1996 survey and from the 1997 tax returns (fiscal year 1996).

Table 2: Household monthly per-capita income before taxes (in Nov 1994 dollars)

Decile	1 **Minimum** US$	2 **Maximum** US$	3 **Average** US$
1	0	37	21
2	37	56	47
3	56	74	65
4	74	95	83
5	95	121	107
6	121	156	137
7	156	201	178
8	201	274	233
9	274	456	351
10	456	55,759	1,022

Table 3: Number of taxpayers by income bracket: IRS data

Bracket (US$)	1 **Tax rate**	2 **IRS**
581 - 1,453	5%	461,028
1,453 - 2,422	13%	113,157
2,422 - 3,390	23%	48,177
3,390 - 4,359	33%	21,535
4,359 - 4,843	35%	6,047
4,843 - 5,812	45%	7,533
larger than 5,812	48%	11,807

further 4,292 taxpayers took advantage of letter (b) allowing them to deduct slightly less than US$14 million from their tax burden. It was not possible to obtain data to allocate the DFL2 allowance among tax payers. However, about 80% of real estate benefits from this allowance, so our calculations assume that *all* property income is exempt from the income tax. Finally, allowances that favor small businesses and unincorporated businesses in particular sectors are not accounted for, because we lack the data to impute them. For this reason, in the calculations that follow we are unable to distinguish between income that was legally not declared and outright tax evasion.

Composition and level of consumption. The consumption patterns of each household was estimated on the basis of the family budget survey (EPF) carried out by the National

Institute of Statistics (INE) in Greater Santiago between December 1987 and November 1988. To estimate the tariffs paid by each household when consuming imported goods, one needs to know what fraction of their expenditure falls on imports and traded goods, not only of final goods, but also inputs used in producing domestically produced final goods. The Input-Output Matrix (IOM) calculated by the Central Bank of Chile for 1986 was used to determine the foreign content of domestically-produced goods, both traded and non-traded. Consumption levels were calculated using the EPF data, assuming that the consumption pattern remained unchanged between 1988 and 1994, growing at the same rate as incomes.

Impact of indirect taxes charged on inputs. Import tariffs and the specific tax on gasoline affect the prices of inputs used in the production of final goods consumed by households.[19] The IOM was used to estimate the impact of indirect taxes on the prices of final goods. For each type of good included in the IOM, coefficients were estimated enabling us to determine what fraction of a households spending on a given type of good corresponds to the indirect payment of a tariff or gasoline tax.

3.4 Incidence assumptions

We assume that direct taxes (business, wage and GC) are paid wholly by the taxed factors, whereas indirect taxes (VAT, tariffs, and excise taxes) are paid entirely by consumers. We also assume that the savings rate is exogenous, and that the fraction of a household's income spent on each good is independent of the tax structure;[20] these are standard assumptions in the literature (see for example Ockner and Pechman [1974, ch.3]).[21] Finally, in the case of Chile it is reasonable to assume that the business tax is not passed on to consumers, because almost all types of businesses are subject to it.

There are two scenarios (at least) under which these assumptions are appropriate and consistent. First, a small open economy where all goods are tradable, and purchasing power parity (PPP) holds. Under such conditions, direct taxes cannot be passed on to consumers because the latter would switch towards imported goods. On the other hand,

[19] The excise tax on gasoline is levied only when gasoline is used as an input of transport services.

[20] The latter assumes that the utility function of the household is Cobb-Douglas.

[21] Shah and Whalley (1991) strongly criticize these assumptions for developing-country studies. They point out that import quotas, price controls and black markets, the fact that income taxes tend to be paid only in cities and corruption associated with tax evasion radically change incidence patterns. However none of these apply to Chile: quotas and price controls are virtually nonexistent; income taxes are also monitored in the rural sector, which is, in any case, small; and corruption in tax administration is, by international standards, low.

if both national and foreign goods are subject to indirect taxes, these will be passed on to consumers. The second scenario is of a closed constant-returns-to-scale economy with Leontief production functions, together with perfectly inelastic factor supplies. In this case any direct tax falls on factors (because their supply is perfectly inelastic) and all indirect taxes are transferred to consumers, because supply prices are determined solely by technology. Note that the national accounts, which we use to estimate the impact of indirect taxes, are constructed on the basis of these assumptions.

Finally, we assume that only profits distributed by firms affect the distribution of income, and that the business tax is paid wholly by those who receive those profits. As has been mentioned already, company profits that are not distributed do not enter our calculation of the income distribution.

3.5 Determination of tax burdens

The distribution of household incomes can be constructed on the basis of income data obtained from the adjusted CASEN survey.[22] The distributional impact of the tax system, and its progressivity, are found by substracting what each household pays in direct and indirect taxes from its incomes. The methodology for calculating the amount of taxes each household pays is described below.[23]

3.5.1 Amount paid in direct taxes

As was mentioned above, income taxes in Chile are personal. Therefore, to estimate the direct tax burden each household bears, we first estimate the direct taxes paid by each individual and then add up these amounts to estimate what is paid by each household. The direct taxes paid by each individual are estimated by deducting from the tax base any income she decides not to declare, as well as any allowances. Thus, to estimate the amount paid in direct taxes by each individual in the CASEN survey, a prior estimate is needed of underdeclared income and tax-free allowances benefiting the individual. This is done following the following steps:

[22] Some income from the CASEN survey is collected on an after-tax basis. For details on how we obtained pre-tax estimates of these incomes see the Methodological Appendix.

[23] In the following discussion we leave out certain details. A detailed description of the calculation methodology is given in the Methodological Appendix.

1. Individuals in the CASEN database whose incomes are high enough to be subject to income tax are separated out.[24] The procedure is then repeated, this time with taxpayers in the SII database. In both cases they are grouped together by centile. (It is worth remembering that it is not possible to identify individuals covered by the CASEN survey in the SII data base, as the tax ID numbers of those surveyed are not recorded). Table 4 shows the number of taxpayers declaring income to the SII by income bracket, and the number that should have declared according to the CASEN survey.

Table 4: Taxpayers by income bracket

	1	2	3
Bracket (US$)	**Tax rate**	**IRS**	**CASEN**
581 - 1,453	5%	461,028	666,531
1,453 - 2,422	13%	113,157	130,555
2,422 - 3,390	23%	44,817	43,929
3,390 - 4,359	33%	21,534	22,252
4,359 - 4,843	35%	6,047	8,594
4,843 - 5,812	45%	7,533	9,870
larger than 5,812	48%	11,807	23,413

2. The number of non-filers is defined as the difference between the number of individuals in the CASEN survey with incomes high enough to be subject to tax, and those individuals who actually filed a declaration with the SII. According to this procedure 239,148 individuals did not file in 1994. To estimate the income of non-filers, a random sample of individuals of size equal to this number was drawn among CASEN individuals. We assumed that individuals chosen by this procedure were the non-filers.[25] Table 5 ranks CASEN individuals according to their taxable (monthly) income bracket. The first two columns show the estimated number of non-filers by income bracket; column 3 shows the amounts underreported.

[24] In addition, individuals subject to tax are divided into GC taxpayers and wage taxpayers, so as to impute the allowances provided by Article 57 bis only to the former.

[25] We assumed that those with annual incomes higher than $15,000 (5% of all taxpayers) always report at least some of their income to the SII. For the rest, the probability of not reporting decreases linearly with income. The distribution was parameterized so that (a) the probability of not filing is zero for incomes equal to $15,000; (b) the expected number of non-filers is equal to the actual number of non-filers.

Table 5: CASEN individuals by income bracket

Income Bracket	1 # Non filers	2 # Filers	3 Amount underreported (MMUS$)	4 Amount reported (MMUS$)	5 3/(3+4)
$581- $ 1,453	239,159	427,372	211	363	0.37
$1,453- $ 2,422	0	130,555	18	219	0.08
$2,422 - $3,390	0	43,929	9	112	0.07
$3,390 - $4,359	0	22,252	9	43	0.18
$4,359 -$ 4,843	0	8,594	5	34	0.13
$4,843 - $5,812	0	9,870	9	43	0.18
larger than $5,812	0	23,413	102	159	0.39

3. To estimate the amount underreported by filers, we ranked individuals from the CASEN survey (i.e., those who were not randomly excluded) by income percentile, and the sum of the incomes in each percentile was compared with the sum of incomes declared to the SII by the equivalent percentile.[26,27] The difference is the amount underreported by that percentile. Within each percentile of the CASEN survey, underreporting is distributed proportionately to the income of each individual. The third and fourth columns show the number of filers and the amount underreported by income bracket. For example, underreporting by individuals with incomes large enough to fall in the top bracket is 39% of income.

4. Tax-free allowances under Article 57 bis., letters (a) and (b) corresponding to each individual in the CASEN survey were estimated in the same way as underreporting, i.e., for each percentile of the CASEN survey the amount of tax-free allowances under Article 57 bis was imputed for the equivalent SII percentile. This amount was distributed among individuals comprising each percentile in proportion to income.

5. Last, the taxable income declared by each individual in the CASEN survey was obtained by subtracting underreporting from their true income. The amount of tax paid by each taxpayer was then obtained by applying the corresponding GC structure of

[26] As mentioned before, this underreporting does not necessarily amount to evasion, because some taxpayers declare less income than their true total income. The CASEN survey does not enable us to identify incomes that do not pay tax for this reason.

[27] The wealthiest decile accounts for 64.7 percent of the total amount underreported

rates and Article 57 bis. allowances.

Once we know the incomes before and after income tax for each individual, it is possible to construct the income distribution at the household level. Since each individual in the CASEN survey belongs to a household, the income of every household, both before and after paying direct taxes, can be found by adding together the incomes of its members. The impact of changes in the tax structure is found by repeating this exercise with new tax parameters.

3.5.2 Amount paid in indirect taxes

The burden of indirect taxes borne by each household is estimated as follows:

1. A tax on transactions can affect the price of final goods both directly (if the final good itself is taxed) and indirectly (if the inputs into the final good are taxed). So, to estimate the effect of indirect taxes on the prices of final goods the IOM input-output coefficients were used. Using this information together with the tax rates applied to different goods, the fraction of the final price corresponding to each of the indirect taxes was calculated for each type of good in the IOM classification.[28]

2. The composition of household expenditure on about 2,000 goods and services was obtained from the family expenditure survey (EPF) at the decile level. These goods were grouped in the 75 categories defined in the IOM. Households covered by the family expenditure survey were ordered by income decile, and for each decile the expenditure of a representative household was calculated. This spending pattern was then assumed to be representative of the expenditure of all households in that decile.

3. The fraction of its income that each household pays in indirect taxes for each good is obtained by adding together, over all IOM goods, the product of (a) the fraction of the final price of each IOM good corresponding to each of the taxes (point 1), and (b) the expenditure of the representative household on each IOM good, expressed as a fraction of its income (point 2). Thus, for the representative household in each decile,we obtain its payments of each indirect tax as a fraction of its income.

Using the results obtained in the previous points, for each household in the CASEN survey the percentage of total expenditure used in paying each indirect tax was estimated.

[28] The details of this procedure and those described in the following points can be found in the Methodological Appendix.

CASEN survey households are grouped in deciles (in the same order as EPF households), and the amount paid in terms of each indirect tax was obtained by adding over the different goods. The impact of changes in the structure of indirect taxes is obtained by repeating this exercise with new parameters.

4 Results

In this section we present the results of a series of exercises we carried out with the model described in the previous section. In Section 4.1 we examine the distributive consequences of the 1994 tax structure. As most other studies, we conclude that the tax structure had virtually no effect on the distribution of income. If anything, the tax structure in force in 1994 was slightly regressive. Section 4.2 suggests that this result is not due to avoidance or evasion: we show that even if all incomes had paid the tax due on them, the distribution would have not changed much. Finally, in Section 4.3 we show the distributional impact of four big changes to the tax structure: the abolition of import tariffs, a rise in VAT from the actual rate of 18% to 25%, a doubling of the rate of the gasoline tax and the replacement of the current progressive income tax with a flat tax. In each case we conclude that the distributional impact is surprisingly small.

4.1 The distribution of the tax burden in 1994

Table 6 shows the distribution of income by deciles in 1994 before any tax is paid. The Gini coefficient is 0.4899, and the ratio between the incomes of the wealthiest and the poorest quintile (henceforth "ratio") is equal to 13.57. It is clear that income is very unequally distributed: the wealthiest quintile receives 56.5% of total income against 4.2% of the poorest quintile.

Column 2 in Table 7 shows the after-tax income distribution (column 3 reproduces Table 6). This is slightly *more* unequal than the pre-tax distribution. The Gini coefficient rises from 0.4899 to 0.4920, and the ratio goes up from 13.57 to 14.13. Column 3, which shows the fraction of income that each decile pays in taxes, suggests that the 1994 tax system marginally worsened the distribution, as it was slightly regressive. Indeed, on average the five poorest deciles paid 17.2% of their incomes in taxes, as against 15.2% of the five wealthiest deciles. The second decile is the group that pays the largest fraction of its income in taxes (18.5%) whereas the ninth decile pays the lowest rate (14.1%).[29]

[29] Note that while the average tax rate paid by the top decile is smaller than the overall average, the

Table 6: Income distribution before taxes

Decile	Income Share (IS)
1	1.35
2	2.81
3	3.78
4	4.59
5	5.75
6	6.76
7	8.39
8	10.11
9	15.22
10	41.23
GINI	0.4899
RATIO	13.57

To understand why the Chilean tax system is slightly regressive, it is useful to look at the last three columns of Table 7 which show the progressivity of the income tax, the value-added tax (VAT), excise taxes (gasoline, jewelry, tobacco, etc.) and import tariffs. On the one hand, the regressivity of the VAT is evident: the highest-income deciles pay a smaller fraction of their incomes in VAT. The only exception is the poorest decile which spends a smaller fraction on VAT than deciles 2, 3 and 4, because a significant part of this group's consumption is not subject to VAT as it constitutes consumption of own production.[30,31]

The income tax, on the other hand, is clearly progressive, yet the revenue it raises is very small.[32] Only the wealthiest decile pays more than 1% of its income in income tax, and even in this case its average rate is only 3.61%, compared with 7% paid in VAT and

after-tax share of the richest decile is *lower* than the pre-tax share. This occurs because after calculating taxes our program reorders households according to their after-tax per capita income. For example, some of the households of the richest decile fall to the ninth decile. This accounts for the fall in the share of the richest decile.

[30] It should be noted that if the only tax was VAT charged at a rate of t percent, a family that consumes all its income in goods subject to tax would pay a fraction equal to $t/(1+t)$ percent in tax. Thus, with VAT at 18 percent, the family would pay 15.25 percent of its income in tax. Nevertheless, since we assume that evasion only benefits producers, the 7 percent rate which appears in Table 7 suggests that only 46 percent of the income of the wealthiest decile pays VAT. This is because the wealthiest decile has a higher savings rate, and several items of consumption, personal services in particular, are not subject to VAT. In fact, the fraction of expenditure not subject to VAT is about 20% for all deciles.

[31] As has been mentioned in the previous section, considering annual incomes probably exaggerates the regressivity of the VAT.

[32] Fontaine and Vergara (1997) emphasize this point.

Table 7: After-tax income distribution

Decile	1 IS pre-tax	2 IS after	3 Progr. Tax Sys.	4 Progr. inc. tax	5 Progr. VAT	6 Progr. other tax
1	1.35	1.28	16.4	0.00	11.2	5.14
2	2.81	2.71	18.5	0.00	12.5	5.94
3	3.78	3.65	17.5	0.00	11.8	5.63
4	4.59	4.63	17.0	0.00	11.5	5.47
5	5.75	5.71	16.5	0.00	11.2	5.36
6	6.76	6.77	16.2	0.02	10.9	5.31
7	8.39	8.33	15.5	0.08	10.3	5.15
8	10.11	10.57	15.0	0.18	9.8	5.10
9	15.22	15.73	14.1	0.62	8.7	4.82
10	41.23	40.63	15.0	3.61	7.0	4.39
GINI	0.4899	0.4920				
RATIO	13.57	14.13				

4.39% paid in other taxes.[33]

4.2 Scenario with neither underreporting nor allowances

The meager revenue performance of the income tax is surprising, although it does coincide with the impression widely held in Chile that "everybody avoids the income tax." An interesting exercise is to calculate whether the income distribution would become more equal if both tax-free allowances and underreporting were completely eliminated. In so far as the assumptions of our model are valid, this exercise sets an upper bound on what the 1994 tax system could have achieved in terms of income distribution.

Table 8 shows the income distribution and progressivity of the income tax when (a) only tax-free allowances are eliminated, and (b) both tax-free allowances and underreporting of income are eliminated. Column 1 shows again the income distribution resulting from the 1994 tax structure, and column 4 shows the progressivity of the corresponding income tax.

The second column of Table 8 shows that the effect of allowances (mainly Article 57 bis,

[33] It is important to remember that our definition of income does not include undistributed profits of investment companies set up to avoid the highest marginal rates of GC tax. If these were included, income would be even more concentrated: since one reason for setting up such a company is to avoid marginal tax rates above 15% (the business tax rate); incomes at this level are only found in the wealthiest decile. Nevertheless, since profits retained in a company pay an average rate of 15%, the progressivity of the income tax would probably increase.

Table 8: Scenario with neither underreporting nor allowances

	1	2	3	4	5	6
Decile	**IS1**	**IS2**	**IS3**	**PIT1**	**PIT2**	**PIT3**
1	1.28	1.29	1.32	0.00	0.00	0.00
2	2.71	2.73	2.80	0.00	0.00	0.00
3	3.65	3.67	3.77	0.00	0.00	0.00
4	4.63	4.65	4.77	0.00	0.00	0.00
5	5.71	5.73	5.88	0.00	0.00	0.01
6	6.77	6.80	6.98	0.02	0.03	0.06
7	8.33	8.36	8.58	0.08	0.10	0.19
8	10.57	10.60	10.84	0.18	0.20	0.38
9	15.73	15.80	16.15	0.62	0.70	1.06
10	40.63	40.38	38.91	3.61	4.07	5.91
Gini	0.4920	0.4901	0.4796			
Ratio	14.13	13.98	13.36			

IS: Income Share

PIT: Progressiveness of the income tax

Scenario 1: Tax system in 1994

Scenario 2: Tax system in 1994 without allowances

Scenario 3: Tax system in 1994, with neither underreporting nor allowances

and DFL2) on the distribution of income is irrelevant: the income share of the wealthiest decile goes down slightly from 40.63% to 40.38%; the progressivity of income tax (column 4) rises marginally. A somewhat greater impact would be achieved by eliminating underreporting, which is significant as we saw in the previous section. The third column of Table 8 shows that, in this case, the share of the wealthiest decile falls to 38.91% of total income. The Gini coefficient improves from 0.4920 to 0.4796, whereas the ratio falls from 14.13 to 13.36. However, while this is the biggest change in income distribution we shall see in this section, it is still far from impressive, especially when one considers that it assumes that income tax evasion is completely eliminated. It is interesting to mention that in this last scenario the decile that most increases its share in the income distribution is the second wealthiest, rising from 15.73 to 16.15%. Any improvement among the poorest deciles is minimal: for example, the poorest decile raises its income share from 1.28% to 1.32%. the share of the wealthiest decile share is the only one that falls, but not by much, from 40.63% to 38.91%.

The last column of Table 8 shows that if both underreporting and tax-free allowances are eliminated, the average tax rate of the wealthiest decile rises from 3.61% to 5.91%. This average rate is low if we consider that in 1994 the top marginal rate was 48% for individuals with monthly incomes over US$6,000. Table 9, which shows the distribution of income within the wealthiest decile, reveals why the revenue potential of the income tax is so small. One needs to get to the 99-th *centile* to find households whose monthly per-capita income is above US$1,250. In other words, a significant fraction of households in the "wealthiest" decile are not so wealthy after all, so the revenue performance of a progressive tax is low.[34]

Table 9: Monthly income distribution: richest decile

	1	2	3
centile	**Minimum** (US$)	**Maximum** (US$)	**Average** (US$)
91	456	503	484
92	504	534	520
93	535	585	560
94	585	634	608
95	635	710	671
96	710	817	764
97	818	943	875
98	945	1,225	1,066
99	1,225	1,715	1,449
100	1,721	55,759	3,207

4.3 Radical modifications of the 1994 tax structure

The results reported above suggest that the 1994 tax structure did not greatly affect the distribution of income. To see whether this conclusion holds for a variety of possible modifications to the tax structure, in this section we explore the distributional impact of four radical changes: (a) increasing the VAT rate from 18% to 25%; (b) abolishing import tariffs; (c) doubling the excise tax rate on gasoline; and (d) substituting a flat tax for the current income tax.

[34] It is important to remember, however, that the distribution figures are calculated without including profits retained by firms. Therefore, the figures we have at our disposal do not allow us to identify income retained by investment companies formed to postpone/avoid/evade the Global Complementary tax. Furthermore, it is important to note that the CASEN survey is not intended to characterize the wealthiest centile, so the probability that the country's really wealthy households are surveyed is low.

Table 10 suggests that the distributive impact of the three first changes are not large (columns 2, 3 and 4). Particularly surprising is the negligible impact caused by the increase in VAT. The Gini coefficient goes up marginally from 0.4920 to 0.4963, and the ratio from 14.13 to 14.47. The wealthiest deciles share of income grows from 40.63% to 40.98%, whereas the shares of the other deciles fall slightly. The distributive impacts of eliminating tariffs or doubling gasoline tax are even smaller (columns 3 and 4), although distribution does improve slightly.

Table 10: Three radical modifications

	1	2	3	4
Decile	**IS1**	**IS4**	**IS5**	**IS6**
1	1.28	1.25	1.29	1.29
2	2.71	2.67	2.72	2.73
3	3.65	3.58	3.66	3.67
4	4.63	4.59	4.63	4.64
5	5.71	5.64	5.72	5.72
6	6.77	6.64	6.76	6.78
7	8.33	8.33	8.34	8.33
8	10.57	10.59	10.54	10.55
9	15.73	15.73	15.77	15.71
10	40.63	40.98	40.58	40.58
GINI	0.4920	0.4963	0.4917	0.4910
RATIO	14.13	14.47	14.13	14.00

IS: Income share 1994.

Scenario 1: Tax system in 1994.

Scenario 4: Tax system in 1994 with 25% VAT.

Scenario 5: Tax system in 1994 without import tariffs.

Scenario 6: Tax system in 1994 doubling the excise rate on gas

The Flat Tax has been under discussion for some time in the United States, its main academic advocates being Hall and Rabushka (1996). The main virtue of this tax is that it supposedly favors saving, apart from allowing considerable simplification in tax administration. However, it is criticized for being regressive. Here we examine the effects of a flat tax that leaves the first US$2,500 of monthly income exempt.[35] The flat tax is interesting because it represents a radical change to the tax in force in 1994 and apparently ought to

[35] A flat tax proposal typically includes an exempt bracket, so as to give it a certain degree of progressivity.

significantly worsen income distribution.[36]

Table 11 shows the distributional impact of a reform of this type. The first and fourth columns once again show income distribution in 1994. Columns 2 and 3 show the distributive impact of the flat tax, in the first case assuming no change in allowances and underreporting, and in the second case assuming that both are completely eliminated. As a flat tax ought to reduce underreporting because it facilitates supervision and makes avoidance less worthwhile, it is to be expected that the actual effect of a reform of this type would fall between the two scenarios considered.

Table 11: Flat-Tax

	1	2	3	4	5	6
Decile	**IS1**	**IS7**	**IS8**	**PIT1**	**PIT7**	**PIT8**
1	1.28	1.26	1.28	0.00	0.00	0.00
2	2.71	2.67	2.71	0.00	0.00	0.00
3	3.65	3.59	3.64	0.00	0.00	0.00
4	4.63	4.55	4.61	0.00	0.00	0.00
5	5.71	5.61	5.68	0.00	0.00	0.00
6	6.77	6.65	6.74	0.02	0.00	0.00
7	8.33	8.19	8.29	0.08	0.00	0.00
8	10.57	10.39	10.53	0.18	0.00	0.00
9	15.73	15.58	15.79	0.62	0.02	0.04
10	40.63	41.50	40.72	3.61	1.58	2.65
GINI	0.4920	0.4985	0.4931			
RATIO	14.13	14.52	14.16			

IS1, PIT1: Income share and progressiveness of income tax. Tax system in 1994
IS7, PIT7: Income share and progressiveness of income tax. Flat-Tax
IS8, PIT8: Previous case with neither underreporting nor allowances

Table 11 suggests that the regressive impact of a flat tax of the type considered is surprisingly small. When the current tax-free allowances are maintained, together with the same level of underreporting, the fraction of income received by the wealthiest decile rises from 40.63% to 41.50%; the Gini coefficient rises slightly from 0.4920 to 0.4985, and the ratio from 14.13 to 14.52. If tax-free allowances and underreporting are eliminated, the income distribution would be almost identical to that obtaining under the 1994 tax

[36]Even though the tax-free bracket mentioned above makes the flat tax progressive, it is less progressive than the current income tax. The tax-exempt bracket was chosen so that no taxpayer ends up paying a higher rate than with the current tax structure.

structure. The wealthiest decile's share rises only marginally, from 40.63% to 40.72%. The final two columns, which show the progressivity of the tax in each case, suggest that tax revenues would fall.

5 Some unpleasant redistributive arithmetic

The results in the previous section suggest that changes in the progressivity of the tax system do not affect the distribution of income significantly, even when quite radical modifications of the tax structure are considered. In this section we perform some simple arithmetic exercises that suggest why this is so. We show that the amount a tax levies has a larger redistributive impact than its progressivity. Furthermore, the difference between both determinants of redistribution is larger when the distribution of incomes before taxation is more unequal.

For the discussion that follows we need some notation. Let λ_i denote the share of income of the i-th decile before taxes and redistribution, t_i denote the average tax rate paid by the i-th decile, and $t \equiv \sum_{j=1}^{10} \lambda_j t_j$ denote the share of total income paid in taxes. Then the share in income of the i-th decile after taxes but before redistribution (this is the measure reported in the tables presented in the previous section) is given by:[37]

$$\lambda_i' \equiv \frac{1-t_i}{1-t}\lambda_i. \tag{1}$$

Let β_i denote the share of government transfers that reaches the i-th decile, with $\sum_i \beta_i = 1$, and assume that a fraction α of income is lost in redistribution. Then the share of income of the i-th decile after taxes and redistribution is given by:

$$\lambda_i'' = \frac{1}{1-\alpha t}[(1-t_i)\lambda_i + (1-\alpha)t\beta_i]. \tag{2}$$

From (1) and (2) it follows that the change in the share of income of the i-th decile is:

$$\lambda_i'' - \lambda_i = \frac{1-\alpha}{1-\alpha t}t\beta_i \; - \; \frac{t_i - \alpha t}{1-\alpha t}\lambda_i, \tag{3}$$

which is equivalent to:

$$\lambda_i'' - \lambda_i = \frac{1}{1-\alpha t}[(t-t_i)\lambda_i + (1-\alpha)(\beta_i - \lambda_i)t]. \tag{4}$$

[37] The expressions that follow assume that no individual changes deciles after paying taxes or after social expenditures take place. The tables in the preceding section do incorporate such changes.

Expression (3) decomposes the redistribution of income that results from the combined effect of taxation and government expenses, into two components. The decile's change in the share of income is the net result of what it receives in expenditures financed through taxation ($\beta_i t$ when $\alpha = 0$) and what it contributes towards financing these expenditures ($\lambda_i t_i$ when $\alpha = 0$).

Two extreme cases can be analyzed based upon (3) and (4), where for simplicity we assume $\alpha = 0$. First consider a poor decile ($\lambda_i \simeq 0$). In this case:

$$\lambda_i'' - \lambda_i \simeq \beta_i t, \tag{5}$$

so that, unless targeting is extremely bad, this decile improves its share of income, benefiting both from an increase in the overall tax burden and from better targeting. The poorest taxpayers care little about the progressivity of the tax system, since their tax burden is small compared to what the receive in any case.

Next consider the richest decile. It follows from (4) that this decile's share in income will decrease more the better targeted government expenditures are (larger $\lambda_{10} - \beta_{10}$), the larger the overall tax burden (larger t), the more progressive the tax rate (larger $t_{10} - t$) and the larger its original share of income (larger λ_{10}).

Table 12: Highly progressive income tax

	1	2	3	4
Decile	λ_i%	t_i%	λ_i'%	λ_i''%
1	1.35	0	1.41	1.82
2	2.81	0	2.93	3.29
3	3.78	0	3.95	4.20
4	4.59	0	4.79	5.02
5	5.75	0	6.00	6.12
6	6.76	0.02	7.06	7.15
7	8.40	0.08	8.77	8.74
8	10.11	0.18	10.54	10.46
9	15.22	0.62	15.80	15.48
10	41.23	10.00	38.75	37.72
Gini	0.4899		0.4720	0.4519
Ratio	13.57		12.56	10.41

Next we consider some arithmetic exercises where we calculate the impact on income distribution of taxes and redistribution. The redistribution coefficients (the β's) in all the exercises that follow are chosen to mimic the actual distribution of transfers in Chile.[38] Table 12 shows the impact of a progressive income tax that generates the same average rates than the income tax in place in 1994, except that it takes 10% of the richest decile's income. This exercise assumes a progressivity of the income tax that is rather unrealistic and extreme: the average rate paid by the richest decile is almost three times the actual rate (3.61%) and almost twice the rate that would attain with no underreporting and tax brakes (5.91%). The column 1 reproduces the before-tax distribution in Chile in 1994. The column 2 shows the tax burden as a percentage of income. Column 3 shows the distribution of income after taxes are levied but before transfers. The column 4 shows the income distribution after taxes and transfers. For the poorest deciles the impact of taxes on their share of income is quite small (see column 4). The reason, quite simply, is that with $t_{10} = 10\%$, $\sum_{j=1}^{10} \lambda_j t_j \equiv t = 4,25\%$, so that

$$\lambda_i' = \frac{\lambda_i}{1 - 0.0425} \cong \lambda_i \times 1.044.$$

In other words, while this progressive income tax decreases the share of the richest decile by almost 2.5 percentage points (from 41.23% to 38.75%), it increases the share of the poorest five deciles by little more than 4.4% of their pre-tax income share. Thus, for example, the share of the poorest decile increases only by 0.06 points, from 1.35% to 1.41%. Levying this tax decreases the ratio from 13.57 to 12.56 and the Gini coefficient from 0.4899 to 0.4720. By contrast, redistribution has a far bigger impact, especially for the three poorest deciles. The share of the poorest decile increases by 0.41 additional percentage points, almost 7 times the impact of the progressive tax. Also, with redistribution the ratio falls to 10.41 and the Gini coefficient to 0.4519. Thus, more than half of the improvement in the income distribution, as measured either by the ratio or the Gini coefficient,[39] is attributable to redistribution, and not to the progressivity of the tax system.

[38] Government expenditures are divided into three categories: pensions (27.7%), transfers (both in money and kind: 37.4%) and general expenses (34.9%). Our measure of autonomous income includes pensions, thus we set $\alpha = 0.277$ thereby ignoring administrative costs and deadweight losses associated with taxation. To determine how well public transfers are targeted, we follow Schkolnik (1993) who finds that the poorest quintile receives 37.5% of all transfers, and the next four quintiles receive, respectively, 28.0%, 19.5%, 11.8% and 3.2%. We assume that each decile receives half of the expenditure of the quintile it belongs to. As to the distribution of general expenses, we assume that they are distributed uniformly across deciles. We report in footnotes how our findings change if we assume that these expenses benefit nobody (thus setting $\alpha = 0.626$). Finally we have that approximately 75% of government expenditures are financed with taxes; we assume that the same fraction of the three kinds of expenditures mentioned above are financed with taxes.

[39] In the case of the ratio it is more than two thirds.

Table 13: Proportional income tax

	1	2	3	4
Decile	$\lambda_i\%$	$t_i\%$	$\lambda_i'\%$	$\lambda_i''\%$
1	1.35	10.00	1.35	2.33
2	2.81	10.00	2.81	3.68
3	3.78	10.00	3.78	4.40
4	4.59	10.00	4.59	5.15
5	5.75	10.00	5.75	6.06
6	6.76	10.00	6.76	6.99
7	8.40	10.00	8.40	8.36
8	10.11	10.00	10.11	9.94
9	15.22	10.00	15.22	14.51
10	41.23	10.00	41.23	38.58
Gini	0.4899		0.4899	0.4404
Ratio	13.57		13.57	8.83

Table 13 shows the effect of a 10% proportional tax. By definition, the after-tax income distribution does not change. The effect of redistribution, on the other hand, is stronger, because the average tax rate is higher. The ratio falls to 8.83 and the Gini coefficient to 0.4404. Thus, a realistic proportional tax improves the distribution of income more than an unrealistically progressive tax.

Analyzing how the share of income of every decile changes after taxation and redistribution can become rather cumbersome, since in the case of intermediate deciles both the decile's tax burden and what it receives in transfers may be significant. This motivates deriving a simple expression for the change in an aggregate measure of inequality.

Proposition 5.1 (Change in the Gini coefficient) *Assume that the absolute tax burden is increasing in before-tax income ($\lambda_i t_i < \lambda_j t_j$, for $i < j$),[40] that the β_i's are decreasing and denote by G_β and $G_{\lambda t}$ the Gini coefficients of the shares of expenditures (the β_i's) and the absolute tax burdens (the $\lambda_i t_i$'s). Denote the Gini coefficients of the income distribution before any taxes and redistribution, and after taxes and redistribution, by G and G'' respectively, and let $\Delta G \equiv G'' - G$. Then:*

$$\Delta G = -\frac{t}{1-\alpha t}\left[(1-\alpha)G_\beta \; + \; G_{\lambda t} \; - \; \alpha G\right]. \tag{6}$$

[40] This assumption holds even if the tax system is somewhat regressive, given the degrees of inequality prevalent in most countries. In particular, it holds by a wide margin with the Chilean data presented in Table 7.

In particular, for $\alpha = 0$:

(7) $$\Delta G = -t[G_{\beta} + G_{\lambda t}].$$

Proof See the Appendix. ∎

It follows directly from (6) that, for a given overall tax burden (value of t), a more progressive tax system (larger value of $G_{\lambda t}$) achieves a larger improvement in the Gini coefficient.

With a proportional tax system we have $G_{\lambda t} = G$. Thus, since most tax systems are not significantly regressive, the scope for improving the distribution of income via increased progressivity is limited to improving $G_{\lambda t}$ from G to 1. In this sense we have that if the initial distribution of income is more unequal, the scope for improving the distribution of income via increased progressivity is smaller. Furthermore, from (7) we see that increasing the progressivity of the tax burden, while the overall tax burden (t) remains fixed, offers less scope for improving the distribution of income than increasing the overall tax burden while the distribution of the tax burden remains unchanged. If the initial Gini is in the neighborhood of 0.5, as is the case for many developing countries, the first strategy can improve the Gini by at most twice as much as the current redistribution of income does, while the second strategy faces no such stringent upper bound.

Table 14: Income distribution after government expenditures: Chile, 1994

	1	2	3	4
Decile	$\lambda_i\%$	t_i	$\lambda_i'\%$	$\lambda_i''\%$
1	1.35	16.4	1.33	2.87
2	2.81	18.5	2.71	4.08
3	3.78	17.5	3.69	4.66
4	4.59	17.0	4.50	5.38
5	5.75	16.5	5.67	6.16
6	6.76	16.2	6.69	7.06
7	8.40	15.5	8.39	8.33
8	10.11	15.0	10.15	9.88
9	15.22	14.1	15.45	14.31
10	41.23	15.0	41.41	37.27
Gini	0.4899		0.4949	0.4170
Ratio	13.57		14.07	7.42

Table 14 shows the distribution of income with the tax structure in place in Chile in 1994, both before taxation and redistribution, and after taxation and redistribution. Even though tax collection is slightly regressive, after government expenses the distribution of income improves considerably: the ratio and Gini fall to 7.42 and 0.4170, respectively.[41] Table 15 decomposes the improvement of the Gini into the contribution of the VAT, the income tax and indirect taxes.[42] Table 15 shows that half the improvement in the income distribution, as measured by the Gini coefficient, is due to the regressive value added tax. The highly progressive income tax accounts for only one fifth of the improvement. This difference is explained by the fact that the VAT levies more than five times the amount levied by the income tax.

Table 15: Decomposition of decrease in Gini coefficient

Tax	Average rate (%)	% of change in Gini
VAT	8.92	49.7
Income	1.61	19.9
Additional	4.86	30.4
TOTAL	15.39	100.0

Since how well expenditure is targeted is unrelated to the progressivity of the tax system, when progressive and proportional taxes are equally costly to levy and cause the same deadweight loss, it is always better to levy a progressive tax. However, in practice indirect taxes are much easier to levy and administer than direct progressive income taxes. Moreover, the main indirect tax levied in Chile, the VAT, is less distortionary than income taxes. In the next section we present a very simple model that enables us to study the determinants of the optimal tax structure when taxes are costly to levy.

6 Inequality and the optimal tax structure

In this section we present a simple model of the optimal determination of the tax structure that incorporates the effectiveness in collecting alternative taxes, their progressivity, the distributional preferences of society and the extent to which government expenditures are targeted. Somewhat surprisingly, we find that proportional taxes are more desirable the more *unequal* the pre-tax income distribution.

[41] If we assume that general government expenses do not benefit anybody, the ratio and Gini fall to 8.66 and 0.4418, respectively.

[42] To obtain this decomposition we compute the change in Gini, after taxation and redistribution, for each tax separately. This decomposition is exact, in the sense that the sum of the changes in Gini is equal to the change when the three taxes are levied simultaneously, when $\alpha = 0$.

6.1 The model

Society consists of two individuals with, respectively, gross incomes of λ_r and λ_p, with $\lambda_r > \lambda_p$ and $\lambda_r + \lambda_p = 1$. The issue is how and how much to redistribute from the rich to the poor individual. We denote the after-tax-and-redistribution income of individual i by d_i and define $d \equiv d_r + d_p$ and $\Delta d \equiv d_r - d_p$. Similarly, $\Delta\lambda \equiv \lambda_r - \lambda_p$.[43]

The government can levy two taxes. The first is a proportional income tax, at rate t_1, that entails collection costs equal to a fraction α_1 of the tax collected. The second is a progressive income tax that is levied only on the rich individual, at rate t_2. This tax entails collection costs equal to a fraction α_2 of the tax collected.[44] The degree to which government expenditures are targeted to the poor individual is captured by the parameter $\beta \in [0, 1]$, the fraction of any government expenditure that accrues to this individual. Last, we assume that the tradeoff between inequality and the deadweight loss of tax collection is captured by a social welfare function that is increasing in the aggregate after-tax income and decreasing in after-tax income inequality. For simplicity we assume this function takes the following form:

$$S(t_1, t_2) \equiv d - \frac{c}{2}(\Delta d)^2, \tag{8}$$

where $c > 0$ captures the degree to which society cares about distributional issues. In the Appendix we show that maximizing this function is (almost) equivalent to maximizing the expected utility of an individual with constant absolute risk aversion coefficient c who ex ante is poor with probability $\frac{1}{2}$.

The following four constants will simplify our presentation:

$$\begin{aligned}
K_1 &\equiv (2\beta - 1)(1 - \alpha_1), \\
K_2 &\equiv 1 + (2\beta - 1)(1 - \alpha_2), \\
L_1 &\equiv \frac{(2\beta - 1) + \Delta\lambda}{(2\beta - 1) + (c\Delta\lambda)^{-1}}, \\
L_2 &\equiv \frac{2\beta c\Delta\lambda}{1 + c\Delta\lambda(2\beta - 1)}.
\end{aligned}$$

The first two constants are measures of the overall effectiveness of tax collection and expenditure targeting in redistributing income to the poor individual. Larger values of either constant are associated with a more efficient system; the largest possible values are 1 for

[43] Note that with two individuals $2\Delta\lambda$ is the Gini coefficient of the before-taxes-and-transfers income distribution.

[44] The α's can also be interpreted as representing the deadweight loss associated with both taxes. For simplicity we assume that they depend neither on the tax rate nor on the amount collected.

K_1 and 2 for K_2. Economic interpretations of L_1 and L_2 are presented shortly. It is also useful to define $M \equiv L_2/L_1$ and to note that:

$$M = \frac{\beta}{\beta - \lambda_p}.$$

6.2 The optimal tax system

We study the problem of choosing a tax system (t_1, t_2) to maximize (8). We are interested in the dependence of the solution on the parameters of the problem: α_1, α_2, β, $\Delta\lambda$ and c. Since collection costs are linear, either none or only one of the taxes will be levied at the optimum, but not both. The following proposition characterizes the optimal tax system.

Proposition 6.1 (The optimal tax system) *Assume that $\beta > \frac{1}{2}$.*[45] *Given the values of β, c and $\Delta\lambda$, Figure 1 divides all possible combinations of α_1 and α_2 into three regions. In region 0, characterized by $\alpha_1 \geq L_1$ and $\alpha_2 \geq L_2$, it is optimal to levy no tax at all. In region 1, characterized by $\alpha_1 \leq L_1$ and $\alpha_2 \geq M\alpha_1$, it is optimal to levy only the proportional tax. The corresponding tax rate is:*

$$t_1^* = \frac{\Delta\lambda}{\Delta\lambda + K_1} - \frac{\alpha_1}{c(\Delta\lambda + K_1)^2}.$$

Finally, in region 2, characterized by $\alpha_2 \leq L_2$ and $\alpha_2 \leq M\alpha_1$, it is optimal to levy only the progressive tax.[46] *The corresponding optimal rate is:*

$$t_2^* = \frac{\Delta\lambda}{\lambda_r K_2} - \frac{\alpha_2}{c\lambda_r K_2^2}.$$

Proof See Appendix. ∎

The first statement in the preposition implies that it may be optimal to levy no tax at all when collection costs are high (large values of α_1 and α_2), society does not care much about inequality (low c), or initial inequality is low (small $\Delta\lambda$).[47] Better targeting of expenditures (an increase in β) implies that a given level of redistribution can be attained with lower taxes. If society does not value redistribution very much (low value of c) this makes levying taxes less attractive.[48] Yet if society values reducing inequality a lot, it

[45] This assumption ensures that L_1 and L_2 in Figure 1 are positive (see below). It may be relaxed at the expense of having to consider three additional figures.

[46] That these three regions define a partition of the unit square follows from the fact that $M = L_2/L_1$.

[47] A straightforward calculation shows that the partial derivatives of the L_i with respect to c and $\Delta\lambda$ are positive.

[48] We have $\partial L_1/\partial\beta > 0$ if and only if $c < (\Delta\lambda)^{-2}$ and $\partial L_2/\partial\beta > 0$ if and only if $c < (\Delta\lambda)^{-1}$.

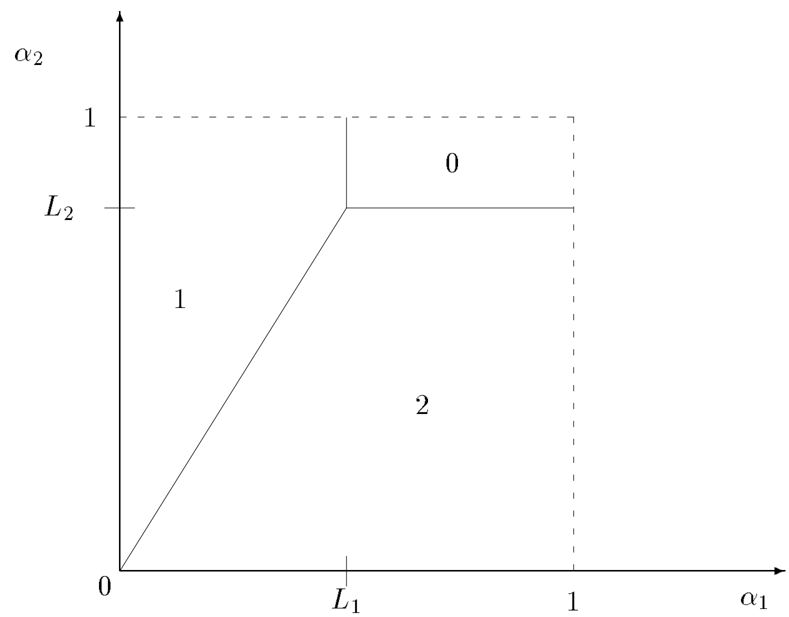


Figure 1:

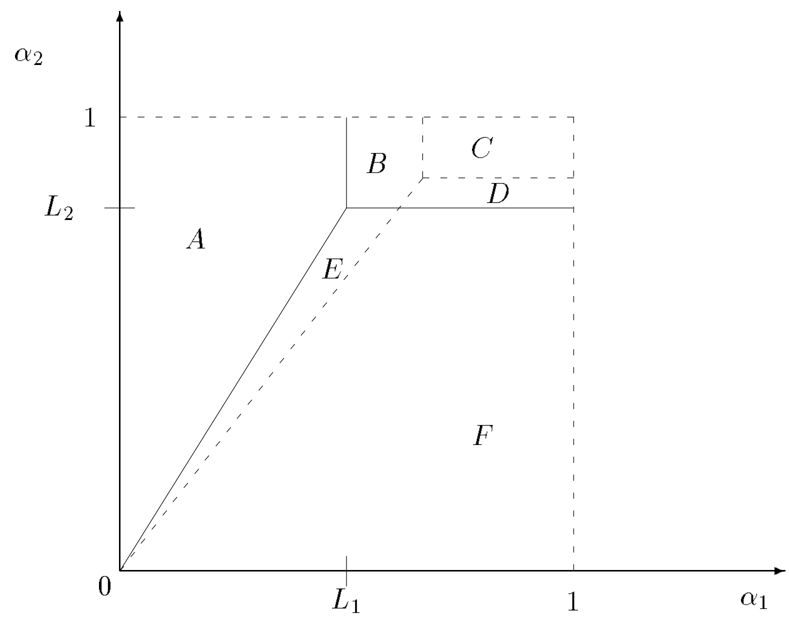
becomes worthwhile to increase taxes after an increase in β.

When redistribution is socially worthwhile, the government chooses the tax that causes a smaller resource loss. If both taxes are equally costly to levy (that is, $\alpha_1 = \alpha_2$) it is always better to levy the progressive tax.[49] The reason is that when taxes are proportional some of the income taken away from the poor individual is wasted ($\lambda_p t_1 \alpha_1$), and another part is redistributed to the rich individual ($[1-\beta](1-\alpha_1)\lambda_p t_1$); both losses are avoided with a progressive tax. Nevertheless, when α_2 is large enough so that it satisfies $\alpha_2 L_1 > \alpha_2 L_2$, it becomes optimal to levy the proportional tax. What is lost when the progressive tax is levied ($\alpha_2 t_2 \lambda_r$) exceeds what is lost levying the proportional tax.

It follows from Proposition 6.1 that it is more likely that levying the proportional tax is optimal the more targeted is expenditure (larger β) and the more *unequal* the initial distribution (larger $\Delta\lambda$).[50] It is quite obvious that the disadvantages of a proportional tax

[49] This follows directly from Figure 1, since the slope of the line joining $(0,0)$ and (L_1, L_2), M, is larger than one.

[50] An increase in $\Delta\lambda$ leads to an increase in L_1 and L_2, and a decrease in M (see Figure 2, where the solid and dashed lines depict the situation before and after the increase in $\Delta\lambda$). It follows that there exists a region in (α_1, α_2)-space where the progressive tax is replaced by the proportional (E), but no region where the proportional tax is substituted by the progressive tax. When β increases we have that the impact on L_1,



	Before	After
A	1	1
B	0	1
C	0	0
D	0	2
E	2	1
F	2	2

Figure 2:

are moderated by adequate targeting, because most of what the poor individual pays in taxes is returned to her. What is somewhat surprising is that a proportional tax is more desirable when the initial distribution is more unequal. The intuition is that a proportional tax takes very little from the poor individual when λ_p is small, so that both $t_1\alpha_1\lambda_p$ and $[1-\beta](1-\alpha_1)\lambda_p t_1$ are small. In the extreme case where $\lambda_p = 0$ the loss is zero. On the other hand, when $\alpha_2 > \alpha_1$ levying the progressive tax is more wasteful. When the initial distribution is very unequal, the incremental waste of resources from levying progressive

L_2 and M depends on whether $c < (\Delta\lambda)^{-1}$, $(\Delta\lambda)^{-1} < c < (\Delta\lambda)^{-2}$ or $c > (\Delta\lambda)^{-2}$. An analysis of the three cases shows that for all of them there exists a region where it is optimal to substitute the progressive tax by the proportional tax, and no region where it is optimal to replace the proportional tax by the progressive tax.

taxes is more important than what is lost when the poor individual pays the proportional tax.

7 Conclusions

To conclude, let us summarize our main findings.

1. The Chilean tax system is slightly regressive. This is the result of the combination of a progressive income tax that exacts little income from the richest decile, and a set of indirect taxes that are mildly regressive but yield much more revenue.

2. Income tax evasion and avoidance is quite large: around 27% of the potential tax base is not reported due to loopholes and evasion. Nevertheless, loopholes and evasion are not responsible for the low yield of the income tax. Completely eliminating them increases the average income tax rate paid by the richest decile from 3,61% to less than 6%.

3. Radical modifications to the tax structure in place in 1994 (e.g., significantly increasing the rate of VAT or substituting a flat tax for the current progressive income tax) have little effect on income distribution.

4. Some simple arithmetic to shows that the scope for directly improving the income distribution via progressive taxes is quite small, the more so the more *unequal* the pre-tax income distribution. Thus, even unrealistically progressive taxes have little direct impact on income distribution at the inequality levels currently prevalent in Chile.

5. The targeting of expenditures and the average tax rate have a far bigger quantitative impact on income distribution. In the case of Chile, once the targeting of expenditures is taken into account, high-yield indirect taxes are responsible for 80.1% of the reduction in income inequality achieved through the redistribution of the taxes considered in this paper. By contrast, the low-yield progressive income tax accounts only for the remaining 19.9% of the reduction.

6. A simple model shows that when progressive taxes are more costly to levy and cause a larger excess burden, broad-based proportional taxes become more desirable. Somewhat surprisingly, the optimal tax system is more biased towards proportional taxes the more *unequal* the pre-tax distribution. Thus, the current tax structure in Chile,

which relies heavily on broad-based indirect taxes like VAT, that are cheap to administer and are generally thought to produce less distortions, is probably closer to the optimum than what is usually thought.

7. The main policy implication of this paper is that the tax structure should be chosen on the basis of tax collection and efficiency criteria, and not according to its redistributive merits. Distributional considerations should enter only when deciding the size of the overall tax burden.

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A Proofs of Section 5

Proof of Proposition 5.1

Consider a sequence $x_1, x_2, \ldots, x_n$ such that $\sum x_i = 1$. If the x_i's are arranged in increasing order, the corresponding Gini coefficient is:

$$G = \frac{2}{n} \sum i x_i - 1 - \frac{1}{n}, \tag{9}$$

while if they are arranged in decreasing order it is:

$$G = 1 + \frac{1}{n} - \frac{2}{n} \sum i x_i. \tag{10}$$

We then have:

$$\begin{aligned}
\Delta G &= \frac{2}{n} \sum_i i(\lambda_i'' - \lambda_i) \\
&= \frac{2}{n} \sum_i i \left\{ \frac{(1-\alpha)t}{1-\alpha t} \beta_i \; - \; \frac{1}{1-\alpha t} \lambda_i t_i \; + \; \frac{\alpha t}{1-\alpha t} \lambda_i \right\} \\
&= \frac{(1-\alpha)t}{1-\alpha t} \left\{ \frac{2}{n} \sum i \beta_i \right\} \; - \; \frac{t}{1-\alpha t} \left\{ \frac{2}{n} \sum i \frac{\lambda_i t_i}{t} \right\} \; + \; \frac{\alpha t}{1-\alpha t} \left\{ \frac{2}{n} \sum i \lambda_i \right\} \\
&= \frac{(1-\alpha)t}{1-\alpha t} \left[1 + \frac{1}{n} - G_\beta \right] \; - \; \frac{t}{1-\alpha t} \left[1 + \frac{1}{n} + G_{\lambda t} \right] \; + \; \frac{\alpha t}{1-\alpha t} \left[1 + \frac{1}{n} + G \right] \\
&= -\frac{t}{1-\alpha t} \left[(1-\alpha) G_\beta + G_{\lambda t} - \alpha G \right].
\end{aligned}$$

In the fourth step we used (9), (10) and the assumptions that the β_i's are decreasing and the $\lambda_i t_i$'s increasing. ∎

B Proofs of Section 6

Proposition B.1 *Consider an individual whose income is equally likely to be d_r or d_p, with $d_r > d_p$. Assume that the individual's utility has a constant coefficient of absolute risk aversion, c. Let $d_{\text{tot}} = d_r + d_p$ and $\Delta d = d_r - d_p$. Then maximizing her expected utility is equivalent to maximizing $d_{\text{tot}} - \frac{c}{2}(\Delta d)^2 + O((\Delta d)^3)$.*

Proof A Taylor expansion of $U(d)$ around $d = \bar{d} \equiv (d_r + d_p)/2$, evaluated at $d = d_r$ and $d = d_p$, leads to:

$$\mathrm{E}[U(d)] = U(\bar{d}) + \frac{1}{4} U''(\bar{d})(\Delta d)^2 + O((\Delta d)^3). \tag{11}$$

Since:

$$U(d) = -\frac{1}{c}e^{-cd},$$

we have that (11) implies that maximizing $\mathrm{E}[U(d)]$ is almost equivalent to maximizing:

$$\mathrm{E}[U(d)] = e^{-c\bar{d}}\left[-\frac{1}{c} - \frac{c}{4}(\Delta d)^2\right]. \tag{12}$$

Maximizing $\mathrm{E}[U(d)]$ is equivalent to minimizing $\log(-\mathrm{E}[U(d)])$, which due to (12) is (almost) equivalent to maximizing:

$$c\bar{d} - \log\left(\frac{1}{c} + \frac{1}{4}c(\Delta d)^2\right). \tag{13}$$

But:

$$\begin{aligned}\log\left(\frac{1}{c} + \frac{1}{4}c(\Delta d)^2\right) &= \log\left(\frac{1}{c}\left[1 + \frac{1}{4}c^2(\Delta d)^2\right]\right)\\ &\simeq -\log(c) + \frac{1}{4}c^2(\Delta d)^2,\end{aligned}$$

where we used the approximation $\log(1+x) \simeq x$ (the error this introduces is of order $(\Delta d)^4$). The last expression and (13) imply that maximizing $\mathrm{E}[U(d)]$ is equivalent, upto a term of order $(\Delta d)^3$, to maximizing $d_{\text{tot}} - \frac{1}{2}c(\Delta d)^2$. ∎

Proof of Proposition 6.1

We have that:

$$\begin{aligned}d_p &= (1-t_1)\lambda_p + \beta[(1-\alpha_1)t_1 + (1-\alpha_2)t_2\lambda_r],\\ d_r &= (1-t_1-t_2)\lambda_r + (1-\beta)[(1-\alpha_1)t_1 + (1-\alpha_2)t_2\lambda_r],\\ d &= 1-\alpha_1 t_1 - \alpha_2 t_2\lambda_r,\\ \Delta d &= (1-t_1)\Delta\lambda - K_2 t_2\lambda_r - K_1 t_1.\end{aligned}$$

Thus, the objective function (8) can be rewritten as:

$$S(t_1,t_2) = 1 - \alpha_1 t_1 - \alpha_2 t_2 \lambda_r - \frac{c}{2}[(1-t_1)\Delta\lambda - K_1 t_1 - K_2 t_2 \lambda_r]^2.$$

The partial derivatives of this function with respect to t_1 and t_2 are:

$$\frac{\partial S}{\partial t_1} = -\alpha_1 + c\left[(1-t_1)\Delta\lambda - K_1 t_1 - K_2 t_2\lambda_r\right](\Delta\lambda + K_1);$$

$$\frac{\partial S}{\partial t_2} = -\alpha_2\lambda_r + c\left[(1-t_1)\Delta\lambda - K_1 t_1 - K_2 t_2 \lambda_r\right] K_2\lambda_r.$$

The Hessian of $S(t_1, t_2)$ can be calculated from the following second partial derivatives:

$$\begin{aligned}
\frac{\partial^2 S}{\partial t_1^2} &= -c(\Delta\lambda + K_1)^2, \\
\frac{\partial^2 S}{\partial t_1 \partial t_2} &= -cK_2\lambda_r(\Delta\lambda + K_1), \\
\frac{\partial^2 S}{\partial t_2^2} &= -cK_2^2\lambda_r^2.
\end{aligned}$$

Since $\partial^2 S/\partial t_1^2$ is negative and the Hessian is equal to zero, it follows that $S(t_1, t_2)$ is concave. Hence:

1. Region 0 is characterized by:

$$\frac{\partial S}{\partial t_1}(0,0) < 0,$$
$$\frac{\partial S}{\partial t_2}(0,0) < 0.$$

2. Region 1 is characterized by:

$$\frac{\partial S}{\partial t_1}(t_1^*, 0) = 0,$$
$$\frac{\partial S}{\partial t_2}(t_1^*, 0) < 0,$$

   with $t_1^* > 0$.

3. Region 2 is characterized by:

$$\frac{\partial S}{\partial t_2}(0, t_2^*) = 0,$$
$$\frac{\partial S}{\partial t_1}(0, t_2^*) < 0,$$

   with $t_2^* > 0$.

Some patient, but straightforward algebra derives the expressions for t_1^* and t_2^* from the characterizations mentioned above, and shows that the three sets of conditions are equivalent to the regions depicted in Figure 1. ∎